

Math 10B with Professor Stankova

Quiz 5; Tuesday, 2/20/2018

Section #203; Time: 930 AM

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Name: \_\_\_\_\_

Circle True or False or leave blank. (1 point for correct answer,  $-1$  for incorrect answer, 0 if left blank)

1. **TRUE** False If  $A, B$  are mutual exclusive events that are independent, then  $P(A) = 0$  or  $P(B) = 0$ .

**Solution:** If  $A, B$  are mutually exclusive, then  $A \cap B = \emptyset$ . Then if they are independent, then  $P(A \cap B) = 0 = P(A)P(B)$  so  $P(A) = 0$  or  $P(B) = 0$ .

2. True **FALSE** If  $A, B$  are independent events and  $B, C$  are independent, then  $A, C$  are independent.

**Solution:** We can take  $A$  and  $C$  to be the same event.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) When rolling a fair 6-sided die, are the events  $A$  that the number rolled is greater than or equal to 3, and  $B$  that the number rolled is odd, independent?

**Solution:** We just need to check  $P(A \cap B) = P(A)P(B)$ . On the left side, the probability is  $\frac{2}{6}$  from having 3, 5, and  $P(A) = \frac{2}{3}$  and  $P(B) = \frac{1}{2}$  so indeed  $P(A \cap B) = P(A)P(B)$ . So they are independent.

- (b) (5 points) There are 10 red and 10 blue balls in a bag. Someone randomly picks out a ball and then places it back and puts 10 more balls of that color into the bag. Then you draw a ball. What is the probability that the 10 balls added were red, given that you drew out a red ball?

**Solution:** We use Bayes Theorem to get

$$\begin{aligned} P(\text{AddRed}|\text{DrawRed}) &= \frac{1}{1 + \frac{P(\text{DrawRed}|\text{AddBlue})P(\text{AddBlue})}{P(\text{DrawRed}|\text{AddRed})P(\text{AddRed})}} \\ &= \frac{1}{1 + \frac{10/30 \cdot 1/2}{20/30 \cdot 1/2}} = \frac{1}{1 + 1/2} = \frac{2}{3}. \end{aligned}$$